Magnetohydrodynamic lubrication flow between parallel rotating disks

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The motion of an electrically conducting, incompressible, viscous fluid in the presence of a magnetic field is analyzed for flow between two parallel disks, one of which rotates at a constant angular velocity. The specific application to liquid metal lubrication in thrust bearings is considered. The two field configurations discussed are: an axial magnetic field with a radial current and a radial magnetic field with an axial current. It is shown that the load capacity of the bearing is dependent on the MHD interactions in the fluid and that the frictional torque on the rotor can be made zero for both field configurations by supplying electrical energy through the electrodes to the fluid.

1. Introduction

The two-dimensional flow of a conducting incompressible fluid in an idealized thrust bearing with applied magnetic and electric fields is of interest for application to high-temperature bearings using liquid-metal lubricants. The interaction of the flowing liquid-metal lubricant with the applied magnetic field can be used to increase the total load which the rotor can support and reduce the viscous drag on the rotor. Because of the cylindrical geometry of the thrust bearing both axial and radial applied magnetic field configurations are possible.

The bearing geometry considered here consists of two plane parallel disks, one of which, the rotor, rotates at an angular speed ω with respect to the second disk or stator. As is usual in a pressurized thrust bearing the stator has a recessed region. However, for simplicity of analysis, the electrode geometries considered here are such that the flow in the recess is not affected by the applied fields. The results obtained could, however, be easily extended to include such effects.

For the case of an axially applied magnetic field the electrodes are concentric cylinders as shown in figure 1. It is assumed that these electrodes are ideal conductors and porous to the fluid flow. Physically, these electrodes represent equipotential surfaces at the step of the recess and exit radius of the bearing. This idealization is permissible because the separation between the rotor and stator is much less than the recess depth. The actual electrodes in a real bearing would probably be located in the recess and at the outer radius of the stator. The stator and rotor are assumed to be non-magnetic insulators and the magnetic field extends only over the region between the electrodes.

In the radial-field geometry the electrodes are the surfaces of the rotor and stator as shown in figure 2. Here it is assumed that the electrodes extend from $r = a$ to $r = b$ and that end effects and the current density in the recess can be

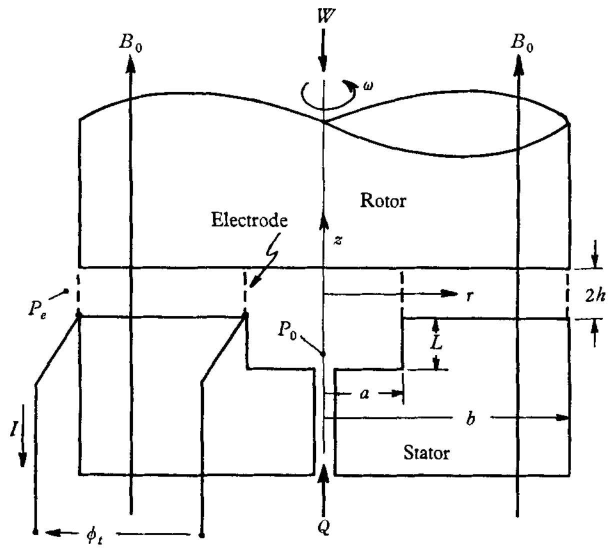

FIGURE 1. The axial-field bearing.

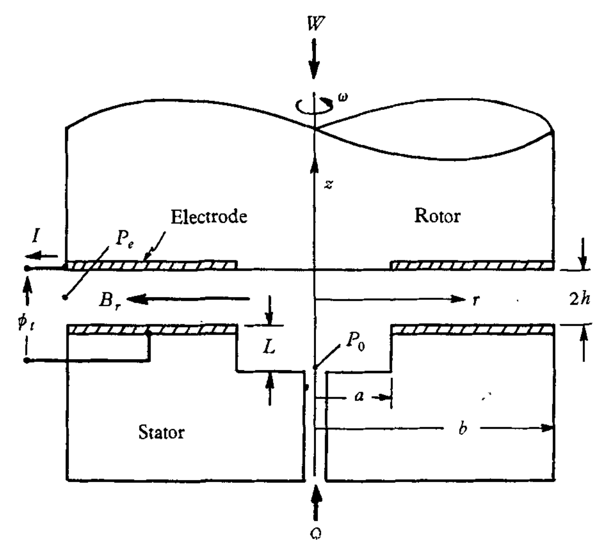

FIGURE 2. The radial-field bearing.

neglected. Again the electrodes are assumed to be ideal conductors and non-magnetic. The radial magnetic field can be obtained in several ways. One radial-field magnet structure is a system of two opposing (opposite exciting currents) magnet coils, one around the stator and the other around the rotor, which are symmetrical about the plane $z = 0$ and which are located on the same axis as the bearing. The radial magnetic field at $z = 0$ is approximately a linearly increasing function of radius and for small film thicknesses the axial component, B_z, is negligible. Another structure which could be used is a coaxial pole magnet with the inner pole embedded in the stator (with the radius of the centre pole less than the recess radius) and the outer pole surrounding the exit region of the bearing. A radial spider with the coils joins the two pole pieces under the stator. For this geometry the radial magnetic field varies approximately as $1/r$ with an axial component only in the recess region near the centre pole piece. It will be seen that the radial-field bearing is not so practical as the former type because of the relatively low pressurization possible.

The flow is considered to be steady-state, incompressible and viscous with constant conductivity σ and viscosity μ. The inertia of the fluid is assumed to be small compared to the viscous forces, which is a valid approximation for the flow considered here (Osterle & Hughes 1958).

2. Basic equations

Maxwell's equations and the magnetic-field constitutive equation can be written (using RMKS units) as

$$\nabla \times \mathbf{E} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{J} = 0, \quad \mathbf{B} = \mu_0 \kappa_m \mathbf{H} \tag{1}$$

and Ohm's law is

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}), \tag{2}$$

where $\mathbf{J}$ is the current density, $\mathbf{E}$ the electric field, $\mathbf{H}$ the magnetic field, μ_0 the permeability of free space, κ_m the relative permeability and $\mathbf{V}$ the fluid velocity. Because the electric field is irrotational ($\nabla \times \mathbf{E} = 0$) a scalar potential ϕ can be defined as

$$-\nabla \phi = \mathbf{E}. \tag{3}$$

The equation of motion, neglecting inertia and electric forces and remembering that the constitutive equations for the magnetic field can be written in any frame of reference for the MHD approximation, can be expressed in vector form as

$$-\nabla P + \mu \nabla^2 \mathbf{V} + \mathbf{J} \times \mathbf{B} - \tfrac{1}{2}\mu_0 H^2 \nabla \kappa_m + \tfrac{1}{2}\mu_0 \nabla(H^2 \rho\, \partial \kappa_m / \partial \rho) = 0, \tag{4}$$

where P and ρ are the pressure and fluid density, respectively. Since the liquid metals are generally non-magnetic the term involving $\nabla \kappa_m$ and the magnetostriction term are neglected. The electric forces are much smaller than the magnetic forces and are neglected throughout. The magnetic fluid could be analyzed without additional difficulty. The $\nabla \kappa_m$ term gives rise to a pressure discontinuity at the fluid interfaces and the magnetostriction term is negligible in the axial-field case except at the fluid interfaces where it contributes to the pressure discontinuity. In the radial-field case the magnetostriction term affects the pressure discontinuity at the fluid interfaces and also has a finite value throughout the fluid. This value can be easily calculated as a function of radius and does not unduly complicate the equation of motion.

Under the present assumptions then, the components of the equation of motion, assuming cylindrical symmetry and lubrication flow ($h \ll b$), are

$$\left.\begin{aligned} r{:}\quad & \frac{\partial P}{\partial r} = \mu\frac{\partial^2 u}{\partial z^2} + (J_\theta B_z - J_z B_\theta),\\ \theta{:}\quad & 0 = \mu\frac{\partial^2 v}{\partial z^2} + (J_z B_r - J_r B_z),\\ z{:}\quad & \frac{\partial P}{\partial z} = (J_r B_\theta - J_\theta B_r), \end{aligned}\right\} \tag{5}$$

where u and v are the radial and tangential velocities, respectively. The continuity equation can be written as

$$Q = \int_{-h}^{h} 2\pi r u\, dz, \tag{6}$$

where Q is the flow rate and h is the half height of the fluid film between the stator and rotor (for $a < r < b$).

3. Axial magnetic field

Referring to figure 1, the external magnetic field B_0 is applied axially in the z-direction, and the electrodes are located at $r = b$ and $r = a$. These electrodes are assumed to be porous and ideal conductors. The induced magnetic fields B_r, B_θ, and induced variations in B_z are assumed to be very small compared to B_0. From $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, an order of magnitude study shows that B_θ is of order $\mu_0 h b \omega B_0$ and B_r of order $\mu_0 h u B_0$. It is only for physically unrealizable velocities of the order of 10^9 m/sec that the induced fields would become comparable to B_0. Because the induced magnetic fields will be small, only the terms in the equation of motion containing B_z need be considered and the equations then become

$$\left.\begin{aligned} r{:}\quad & \frac{\partial P}{\partial r} = \mu\frac{\partial^2 u}{\partial z^2} + J_\theta B_0,\\ \theta{:}\quad & 0 = \mu\frac{\partial^2 v}{\partial z^2} - J_r B_0,\\ z{:}\quad & \frac{\partial P}{\partial z} = 0, \end{aligned}\right\} \tag{7}$$

where the z-variation in pressure is negligible.

From $\nabla \times \mathbf{E} = 0$ and the assumption of cylindrical symmetry, it follows that

$$\frac{\partial E_\theta}{\partial z} = 0, \quad \frac{1}{r}\frac{\partial}{\partial r}(r E_\theta) = 0. \tag{8}$$

Hence E_θ must be of the form $1/r$, but, since E_θ is zero at the inner and outer electrodes, E_θ must be zero everywhere. Substituting for J_θ from equation (2) into the r-part of equation (7) results in the following differential equation for the radial velocity u:

$$\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\mu} u = \frac{1}{\mu}\frac{\partial P}{\partial r}, \tag{9}$$

where $\partial P/\partial r$ is a function of r only. Using the boundary conditions that $u = 0$ at $z = \pm h$, the solution to equation (9) is

$$u = \frac{h^2}{M^2\mu}\frac{\partial P}{\partial r}\left(\frac{\cosh Mz/h}{\cosh M} - 1\right), \tag{10}$$

where M is the Hartmann number $(\sigma h^2 B_0^2/\mu)^{\frac{1}{2}}$.

From the continuity equation (6) the flow rate is

$$Q = \frac{4\pi r h^3}{\mu M^3}\frac{\partial P}{\partial r}(\tanh M - M). \tag{11}$$

Then using the condition that $P = P_e$ at $r = b$ and $P = P_0$ at $r = a$, integration of equation (11) over r gives the flow rate

$$Q = \frac{4\pi h^3 (P_0 - P_e)}{\mu \ln (b/a)}\left(\frac{M - \tanh M}{M^3}\right) = 3Q_0\left(\frac{M - \tanh M}{M^3}\right), \tag{12}$$

where Q_0 is the flow rate for zero Hartmann number (no applied field) and the pressure distribution is

$$P_0 - P = \frac{(P_0 - P_e)\ln (r/a)}{\ln (b/a)}. \tag{13}$$

The radial velocity u is then determined from (10) and (13) as

$$u = \frac{h^2(P_0 - P_e)}{\mu M^2 \ln (b/a)}\frac{1}{r}\left(1 - \frac{\cosh (Mz/h)}{\cosh M}\right). \tag{14}$$

For a fixed $(P_0 - P_e)$, the pressure distribution given by (13) is the same as that obtained when no fields are present and it is evident that only the flow rate and radial velocity profile are affected by the application of an axial magnetic field. In the recess region, u can be neglected and the recess is essentially at a constant pressure P_0 which is the pump supply pressure. The fluid flow rate is decreased by the application of the magnetic field which implies that in order to maintain a given pressure P_0 or a given pressure load on the rotor, less pump work is necessary. The normalized flow rate as a function of the Hartmann number M, for a fixed pressure difference $(P_0 - P_e)$, is shown in figure 3. Conversely, if the flow rate is held constant, the recess pressure and load increase with increasing M as shown in figure 4. The relations for u, P, and Q are independent of the external electrical characteristics.

The total pressure load W of the bearing can be expressed as

$$W = \int_a^b (P - P_e)\, 2\pi r\, dr + \pi a^2 (P_0 - P_e). \tag{15}$$

Substituting for P and integrating gives

$$W = \frac{\pi(b^2 - a^2)(P_0 - P_e)}{2\ln (b/a)} = \frac{Q(b^2 - a^2)\mu M^3}{8h^3(M - \tanh M)} = \frac{W_0}{3}\frac{M^3}{(M - \tanh M)}, \tag{16}$$

where W_0 is defined by the above equation.

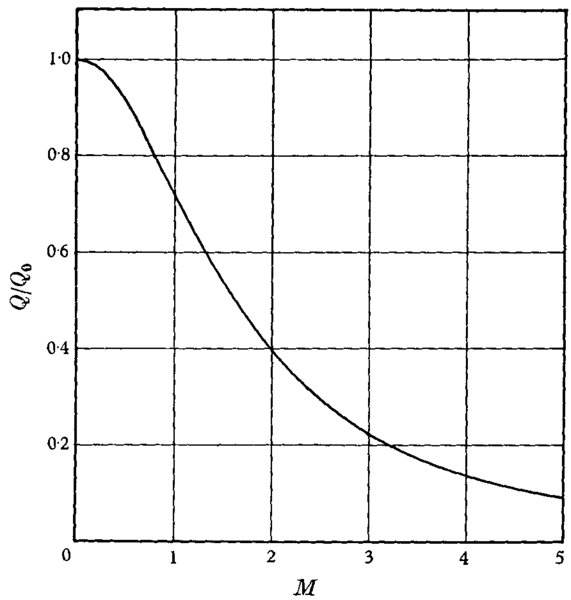

FIGURE 3. Plot of the normalized flow rate *vs* Hartmann number M for the axial-field geometry, the load W (and pressure difference $P_0 - P_e$) being held constant.

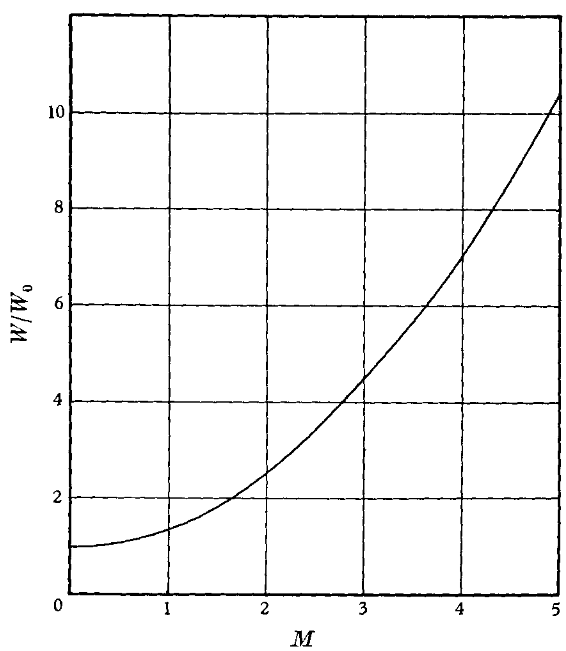

FIGURE 4. Plot of the normalized load W/W_0 as a function of the Hartmann number M for the axial-field geometry, the flow rate Q being held constant.

The tangential velocity v can now be found. From equation (3) the potential between the electrodes is defined as

$$\phi_t = -\int_a^b E_r dr, \tag{17}$$

where ϕ_t is the terminal voltage, a constant. Then, because J_z is of order $\sigma v B_r$ or $\sigma u B_\theta$, it is negligible compared to J_r which is of order $\sigma v B_0$. From the equations $\nabla \times \mathbf{E} = 0$ and $\nabla \cdot \mathbf{J} = 0$, we find that

$$\frac{1}{r}\frac{\partial}{\partial r}(r J_r) + \frac{\partial J_z}{\partial z} = 0, \quad \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = 0. \tag{18}$$

Since $h \ll b$, the first equation is not useful even though $J_z \ll J_r$. However E_r is of order J_r/σ and E_z is of order J_z/σ so that $E_r \gg E_z$. The maximum change in E_z over r is of order E_z and hence the maximum change in E_r over z is of order hE_z/b which is negligible compared to E_r and it may be concluded that E_r is essentially a function of r only.

Substituting for J_r into the θ-part of equation (5), the equation for the tangential velocity becomes

$$0 = \mu \frac{\partial^2 v}{\partial z^2} - \sigma(E_r + vB_0)\, B_0 \tag{19}$$

which can be integrated immediately, using the boundary conditions $v = 0$ at $z = -h$ and $v = r\omega$ at $z = h$, and remembering that E_r is a function of r only, to give

$$v = \frac{E_r}{B_0}\left(\frac{\cosh(Mz/h)}{\cosh M} - 1\right) + \frac{r\omega}{2}\left(\frac{\cosh(Mz/h)}{\cosh M} + \frac{\sinh(Mz/h)}{\sinh M}\right). \tag{20}$$

To determine $E_r(r)$, the expression for v can be combined with Ohm's law and the radial current density integrated over the surface of the cylinder whose area is $4\pi rh$ to find the total current as

$$I = \int_{-h}^{h} 2\pi r\sigma(E_r + vB_0)\, dz. \tag{21}$$

The radial electric field is, in terms of the total current,

$$E_r = \frac{IM}{4\pi\sigma h \tanh M}\frac{1}{r} - \tfrac{1}{2} r\omega B_0. \tag{22}$$

Then the tangential velocity is, in terms of I,

$$v = \frac{I}{4\pi(\sigma\mu)^{\frac{1}{2}} \tanh M}\frac{1}{r}\left(\frac{\cosh(Mz/h)}{\cosh M} - 1\right) + \tfrac{1}{2} r\omega\left(1 + \frac{\sinh(Mz/h)}{\sinh M}\right). \tag{23}$$

From equations (17) and (22), the terminal voltage ϕ_t is

$$\phi_t = \frac{-IM}{4\pi\sigma h \tanh M}\ln(b/a) + \tfrac{1}{4}\omega B_0 (b^2 - a^2), \tag{24}$$

so that the open circuit voltage $\phi_{t\,\mathrm{o.c.}}$ is

$$\phi_{t\,\mathrm{o.c.}} = \tfrac{1}{4}\omega B_0 (b^2 - a^2), \tag{25}$$

and the short circuit current $I_{s.c.}$ is

$$I_{s.c.} = \frac{\omega B_0 \pi \sigma h (b^2 - a^2) \tanh M}{M \ln (b/a)}. \tag{26}$$

Then the internal resistance, which is the ratio of $\phi_{t\,o.c.}$ to $I_{s.c.}$, becomes

$$R_i = \frac{M \ln (b/a)}{4\pi \sigma h \tanh M}. \tag{27}$$

In the recess region ($a > r > 0$), the current density is very small and the applied magnetic field is essentially zero. Therefore the tangential component of the equation of motion in the recess is

$$\mu \partial^2 v / \partial z^2 = 0, \tag{28}$$

which has the solution (for boundary conditions $v = 0$ at $z = -L$ and $v = r\omega$ at $z = h$)

$$v = \frac{r\omega(z+h+L)}{(2h+L)}, \tag{29}$$

where L is the depth of the recess from the surface of the stator plate.

The total drag torque on the rotor can now be computed from

$$T = \int_0^b \left(\frac{\partial v}{\partial z}\right)_{z=h} 2\pi \mu r^2 dr. \tag{30}$$

Using equations (23) and (29), the drag torque T is

$$T = \frac{\mu \omega \pi a^4}{2(2h+L)} + \frac{IM(b^2-a^2)}{4h}\left(\frac{\mu}{\sigma}\right)^{\frac{1}{2}} + \frac{\mu M \omega \pi (b^4 - a^4)}{4h} \coth M, \tag{31}$$

where the first term represents the torque in the region of the recess. From equation (31) it can be seen that in order to make the drag go to zero the current I must be negative. This is confirmed by simple energy considerations because, when I is negative, power is supplied to the bearing from an external source. This power constitutes the power which is dissipated in the fluid as Joule heat and the power to provide the pumping action on the fluid. The value of I' for zero torque, $T = 0$, is

$$I' = \frac{-\pi\omega(\sigma\mu)^{\frac{1}{2}}\,[2ha^4 + (2h+L)\,M(b^4-a^4)\coth M]}{(2h+L)\,(b^2-a^2)\,M}. \tag{32}$$

If $b \gg a$ this current can be approximated by

$$I' = -\pi\omega(\sigma\mu)^{\frac{1}{2}}\, b^2 \coth M \tag{33}$$

and for large values of the Hartmann number the current approaches the value $-\pi\omega b^2(\sigma\mu)^{\frac{1}{2}}$.

The radial current density in the fluid can now be found from Ohm's law and equation (23) which give

$$J_r = \frac{IM}{4\pi h}\frac{1}{r}\frac{\cosh(\mu z/h)}{\sinh M} + \frac{(\sigma\mu)^{\frac{1}{2}} M r\omega}{2h}\frac{\sinh(Mz/h)}{\sinh M}. \tag{34}$$

Then from $\nabla \cdot \mathbf{J} = 0$, the axial current density J_z is

$$J_z = \omega(\sigma\mu)^{\frac{1}{2}}\left[\coth M - \frac{\cosh(Mz/h)}{\sinh M}\right]. \tag{35}$$

Since E_θ is zero, J_θ is simply $-\sigma u B_0$ which, like J_z, is independent of the terminal (electrical) characteristics of the bearing. J_z and J_θ go to zero at the surface of the rotor and stator plates so that circulating currents are set up within the fluid. The above functions for the current densities completely determine these circulating currents.

4. Radial magnetic field

The geometry for the radially applied magnetic field bearing is shown in figure 2 where the electrodes are assumed to be ideal conductors located at $z = \pm h$. The applied radial field is assumed to be a function of r only. By the same argument as in § 3 the induced magnetic fields are neglected and the tangential component of the equation of motion becomes

$$0 = \mu\frac{\partial^2 v}{\partial z^2} + J_z B_r. \tag{36}$$

The exact form of $B_r(r)$ is not yet specified in order to preserve as much generality as possible. Actually, of course, the only function B_r that is not a function of z must be of the form $1/r$. However, since the plate spacing is small other r variations can be constructed that have only a negligible value of B_z over the fluid.

Using again an order of magnitude approximation, the equation $\nabla \cdot \mathbf{J} = 0$ implies that J_z is a function of r only. This conclusion is reached because J_r is of order $\sigma v B_z$ and J_z is of order $\sigma v B_r$ which gives $J_z \gg J_r$. Then from the relative magnitudes of the derivatives $\partial J_r/\partial r$ and $\partial J_z/\partial z$ in the equation $\nabla \cdot \mathbf{J} = 0$ it can be seen by an argument similar to that for the electric fields in § 3 that J_z is essentially a function of r only.

Equation (36) can be integrated directly, using the boundary conditions on v, to give

$$v = \frac{\frac{1}{2}J_z B_r}{\mu}(h^2 - z^2) + \tfrac{1}{2}r\omega\left(1 + \frac{z}{h}\right). \tag{37}$$

Substituting the result for v into Ohm's law gives

$$J_z = \sigma[E_z - (1/2\mu)\, J_z B_r^2(h^2 - z^2) - \tfrac{1}{2}r\omega B_r(1 + z/h)]. \tag{38}$$

Then solving the above equation for E_z and integrating over z, the current density J_z can be determined as a function of the magnetic field strength and the terminal voltage ϕ_t which is now defined as

$$\phi_t = -\int_{-h}^{h} E_z dz. \tag{39}$$

There results then

$$J_z = \frac{-3\sigma\mu(\phi_t + r\omega h B_r)}{6h\mu + 2B_r^2 h^3\sigma}. \tag{40}$$

The total current I can then be related to the terminal voltage ϕ_t by evaluating the integral

$$I = \int_a^b 2\pi J_z r\,dr \tag{41}$$

for a given B_r.

If the current density in the region of the recess is zero (assuming the annular type of electrode which does not extend over the recess) the radial equation of motion and velocity are the same as those given by equations (28) and (29). The total frictional drag on the rotor can then be evaluated by performing the integration indicated by equation (30) for a given function B_r.

The tangential velocity profile and drag will now be found for an axial magnetic field of the form

$$B_r = \frac{B_0 b}{r}, \tag{42}$$

which is easily obtainable in practice. B_0 is the value of the field at $r = b$. Then from equation (40) the axial current density is

$$J_z = \frac{-3\sigma\mu r^2}{2h}\left[\frac{\phi_t + \omega B_0 bh}{3\mu r^2 + \sigma B_0^2 b^2 h^2}\right] \tag{43}$$

and from (41) the total current is

$$I = \frac{-\sigma\pi}{2h}(\phi_t + \omega B_0 bh)\left[b^2 - a^2 - \frac{\sigma B_0^2 b^2 h^2}{3\mu}\ln\left(\frac{3\mu b^2 + \sigma B_0^2 b^2 h^2}{3\mu a^2 + \sigma B_0^2 b^2 h^2}\right)\right]. \tag{44}$$

The tangential velocity v then, from equation (37), is

$$v = \frac{-3\sigma B_0 b}{4h}\left[\frac{\phi_t + \omega B_0 bh}{3\mu r^2 + \sigma B_0^2 b^2 h^2}\right](h^2 - z^2)\,r + \tfrac{1}{2} r\omega\left(1 + \frac{z}{h}\right). \tag{45}$$

Evaluating $\partial v/\partial z$ at $z = h$ and using equation (30), the drag torque becomes

$$T = -B_0 bhI + \mu\pi\omega\left(\frac{(b^4 - a^4)}{4h} + \frac{a^4}{2(2h + L)}\right). \tag{46}$$

Setting $T = 0$, the required current for zero drag is

$$I' = \frac{\mu\pi\omega}{B_0 bh}\left[\frac{(b^4 - a^4)}{4h} + \frac{a^4}{2(2h + L)}\right]. \tag{47}$$

If $b \gg a$, I' can be approximated by

$$I' \approx \mu\pi\omega b^3/4B_0 h^2. \tag{48}$$

It is seen that the current for zero drag is linearly related to the angular velocity of the rotor.

The radial velocity and pressure distribution can now be determined. Referring to the radial equation of motion (5) it is evident that the pressure gradient will be affected primarily by the interaction of J_z and B_θ since the other body-force term, $J_\theta B_z$, is negligible. Then the r-component in equation (5) becomes

$$\frac{\partial P}{\partial r} = \mu\frac{\partial^2 u}{\partial z^2} - J_z B_\theta. \tag{49}$$

However, from the equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ it can be seen that the magnitude of B_θ is of order $\mu_0 J_z$, where μ_0 is the magnetic permeability, and therefore the magnitude of the $J_z B_\theta$ force will be several orders of magnitude smaller than the pressure or viscous forces. This observation indicates that the pressurization due to the electromagnetic body force will be essentially negligible for this radial-field geometry except for very large values of J_z which may be difficult to obtain in practice. Hence the pressure distribution and load are essentially unaffected by the MHD effects, but for the sake of completeness they are considered in this analysis.

From the equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, the tangential component of the magnetic field B_θ is

$$B_\theta = \frac{\mu_0}{r} \int_a^r J_z r \, dr, \tag{50}$$

where J_z is determined from equation (40). Now, since $J_z B_\theta$ is only a function of r and is determined explicitly for any given function B_r, equation (49) can be integrated with the boundary conditions $u = 0$ at $z = \pm h$ to give

$$u = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial r} + J_z B_\theta\right)(h^2 - z^2). \tag{51}$$

From continuity (equation (6)), the flow rate is

$$Q = -\frac{4\pi h^3}{3\mu}\left(\frac{\partial P}{\partial r} + J_z B_\theta\right) r. \tag{52}$$

Integrating (52) for P, using the boundary condition that $P = P_e$ at $r = b$ and $P = P_0$ at $r = a$, gives

$$Q = \frac{4\pi h^3}{3\mu \ln b/a}\left(P_0 - P_e - \int_a^b J_z B_\theta \, dr\right) \tag{53}$$

and

$$P - P_e = \frac{(P_0 - P_e)\ln(b/r)}{\ln(b/a)} - \left(\int_a^b J_z B_\theta \, dr\right)\frac{\ln(b/r)}{\ln(b/a)} + \int_r^b J_z B_\theta \, dr. \tag{54}$$

Then u is given by

$$u = \frac{(h^2 - z^2)}{2\mu r \ln(b/a)}\left\{P_0 - P_e - \int_a^b J_z B_\theta \, dr\right\}. \tag{55}$$

The first term of equation (54) is the usual pressure distribution and the second and third terms represent a 'pinch' pressure that can pressurize the bearing even in the absence of flow (no external pump). However, as was pointed out earlier this pressure term is very small except for extremely large values of J_z. Physically this electromagnetic pressure term is the so-called 'pinch' pressure.

5. Conclusions

An analysis has been presented for two magnetic field geometries which are most easily obtainable in practice. For the case of an applied axial field the pressure distribution is identical with that obtained without a magnetic field. However, the flow rate Q is a function of the Hartmann number and for a fixed pressure difference across the bearing (equivalent to a fixed load) Q decreases with increasing Hartmann number (figure 3). The bearing is then essentially pressurized because a constant inlet pressure P_0 can be maintained with less pressurizing pump work as the Hartmann number is increased. However, as Q approaches zero, the pressure becomes zero for any finite value of the Hartmann number. It should also be pointed out that the flow rate as a function of the Hartmann number is independent of the electrical loading of the electrodes.

The pressure distribution in the radial-field bearing on the other hand can be directly affected by the electromagnetic effects, even when the flow rate is zero. However, this pressurization is negligible from a practical point of view because it is of the order of magnitude of the 'pinch' pressure which requires extremely high currents to achieve useful pressure. Unlike the axial-field case this pressure is dependent on the electrical loading characteristics of the electrodes.

The viscous frictional drag torque on the rotor can be made zero for both geometries by supplying electrical power from an external source so that the fluid is pumped along with the rotor. For both cases the current required to maintain zero drag is a linear function of the rotor angular velocity. If the power input is increased above that required to maintain zero drag the device acts as a motor and energy is supplied to the rotor. When an electrical load is connected to the electrodes energy can be removed and the bearing acts as a generator. For open circuit conditions, $I = 0$, the drag on the rotor is greater than the corresponding drag for the non-MHD bearing (for a given pressure load) since the Joule heating losses must be supplied by the mechanical power through the rotor. The efficiency of the device in any particular mode of operation can be found by considering the two dissipation effects, Joule heating and viscous shear.

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